Pre-Calculus/Trig 3 Name: KEY
UNIT 2: Polynomial Functions – REVIEW Date: ____________

POLYNOMIAL FUNCTIONS REVIEW

Directions: Divide each polynomial.

1.) $(5x^3 - 6x^2 + 8) \div (x - 4)$

$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 8 \\ & & 20 & -56 & -224 \\ \hline & 5 & 14 & 56 & 232 \end{array}$$

$5x^2 + 4x + 56 + \frac{232}{x-4}$

2.) $(x^4 + 5x^3 + 6x^2 - x - 2) \div (x^2 - 3x + 2)$

$x^2 + 8x + 28 + \frac{67x-56}{x^2-3x+2}$

$$\begin{array}{r|l} x^2-3x+2 & x^4 + 5x^3 + 6x^2 - x - 2 \\ & -(x^4 - 3x^3 + 2x^2) \\ & \quad 8x^3 + 4x^2 - x \\ & -(8x^3 - 24x^2 + 16x) \\ & \quad 28x^2 - 15x - 2 \\ & -(28x^2 - 84x + 56) \\ & \quad 67x - 56 \end{array}$$

Directions: State the end behavior of each function.

3.) $f(x) = 5 - 2x - 3x^2$

$x \to -\infty \quad f(x) \to$ $-\infty$

$x \to \infty \quad f(x) \to$ $-\infty$

4.) $f(x) = 2x^5 - 5x + 7$

$x \to -\infty \quad f(x) \to$ $-\infty$

$x \to \infty \quad f(x) \to$ ∞

5.) $f(x) = -x^3 + 2x^2 + 3x - 4$

$x \to -\infty \quad f(x) \to$ ∞

$x \to \infty \quad f(x) \to$ $-\infty$

Directions: Determine whether the function is even, odd, or neither. Then describe the symmetry.

6.) $f(x) = x^4 - x^2 + 4$

$f(-x) = x^4 - x^2 + 4$

EVEN → Y-AXIS SYM.

7.) $f(x) = x^3 - x - 2$

$f(-x) = -x^3 + x - 2$

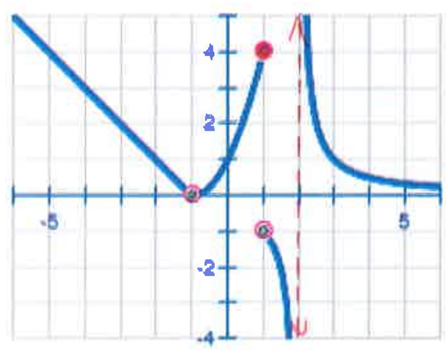
NEITHER

8.) $f(x) = x^3 - x$

$f(-x) = -x^3 + x$

ODD → ORIGIN SYM.

Directions: State the increasing, decreasing and constant intervals in interval notation.



9.) INCREASING: $(-1, 1) \cup$

10.) DECREASING: $(-\infty, -1) \cup (1, 2) \cup (2, \infty)$

11.) CONSTANT: NONE

Directions: Use the Remainder Theorem to determine whether $(x + 4)$ is a factor of the function below.

12.) $f(x) = x^3 - x^2 - 24x - 56$

$$\begin{array}{r|rrrr} -4 & 1 & -1 & -24 & -56 \\ & & -4 & 20 & 16 \\ \hline & 1 & -5 & -4 & -40 \end{array}$$

NOT A FACTOR!

Directions: Use synthetic division to find the remainder. Is the divisor a factor of the polynomial?

13.) $(2x^4 + 14x^3 - 2x^2 - 14x) \div (x + 7)$

$$\begin{array}{r|rrrrr} -7 & 2 & 14 & -2 & -14 & 0 \\ & & -14 & 0 & 14 & 0 \\ \hline & 2 & 0 & -2 & 0 & 0 \end{array}$$

$(x+7)$ IS A FACTOR

14.) $(3x^3 - 17x^2 + 15x - 25) \div (x + 5)$

$$\begin{array}{r|rrrr} -5 & 3 & -17 & 15 & -25 \\ & & -15 & 160 & 725 \\ \hline & 3 & -32 & -145 & -700 \end{array}$$

$(x+5)$ IS NOT A FACTOR

Directions: Write a polynomial function with the given zeros that has the least degree and all real coefficients.

15.) $x = -3, x = 2, x = \frac{1}{2}$

$f(x) = (x+3)(x-2)(2x-1)$

$= (x+3)(2x^2-5x+2)$

$= 2x^3 - 5x^2 + 2x + 6x^2 - 15x + 6$

$= \boxed{2x^3 + x^2 - 13x + 6}$

16.) $x = 2, x = 4i$, $x = -4i$!

$f(x) = (x-2)(x-4i)(x+4i)$

$= (x-2)(x^2+16)$

$= x^3 + 16x - 2x^2 - 32$

$= \boxed{x^3 - 2x^2 + 16x - 32}$

17.) $x = -5$ (multiplicity of 2)

$f(x) = (x+5)(x+5)$

$= \boxed{x^2 + 10x + 25}$

Directions: Given the description of each polynomial function, answer the questions.

18.) The graph of a polynomial function $f(x)$ has a root of $x = -2$ (multiplicity of 3)

a.) Does the graph cross or touch the x-axis at $x = -2$? CROSS →

ODD MULT → CROSS !
EVEN MULT → TOUCH !

b.) What is the least degree of $f(x)$? THIRD DEGREE

c.) Write $f(x)$ with real coefficients and with the least degree.

$f(x) = (x+2)(x+2)(x+2)$

$f(x) = (x+2)(x^2+4x+4)$

$f(x) = x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$

$\boxed{f(x) = x^3 + 6x^2 + 12x + 8}$

19.) The graph of a polynomial function $f(x)$ has a root at $x = -4$ (multiplicity of 2) and a root at $x = -3i$. , $x = 3i$!

a.) Does the graph cross or touch the x-axis at $x = -4$? TOUCH

b.) Does the graph cross or touch the x-axis at $x = -3i$? NEITHER → IMAGINARY!

c.) What is the least degree of $f(x)$? FOURTH DEGREE

d.) Write $f(x)$ with real coefficients and with the least degree.

$f(x) = (x+4)(x+4)(x+3i)(x-3i)$

$f(x) = (x^2+8x+16)(x^2+9)$

$f(x) = x^4 + 9x^2 + 8x^3 + 72x + 16x^2 + 144$

$\boxed{f(x) = x^4 + 8x^3 + 25x^2 + 72x + 144}$

Directions: Use Descartes' Rule of Signs to determine the possible amount of positive and negative real zeros.

20.) $f(x) = 4x^2 - 8x + 3$

$f(x)$ → + − + 2 or 0 REAL'S POS.

$f(-x)$ → + + + 0 NEGATIVE

21.) $g(x) = -5x^3 + x^2 - x + 5$

$g(x)$ → − + − + 3 or 1 POSITIVE

$g(-x)$ → + + + + 0 NEGATIVE

22.) $j(x) = 3x^4 + 2x^2 + x + 3$

$j(x)$ → + + + + 0 POSITIVE

$j(-x)$ → + + − + 2 or 0 NEGATIVE

Directions: Use Synthetic Division to determine whether each value is an upper bound, lower bound, a zero or neither.

23.) $f(x) = x^4 - 4x^3 + 16x - 16$

a.) $x = -4$ LOWER BOUND

b.) $x = 4$ UPPER BOUND

c.) $x = 2$ ZERO

d. $x = -1$ NEITHER

$$\begin{array}{r|rrrrr} -4 & 1 & -4 & 0 & 16 & -16 \\ & & -4 & 32 & -128 & 448 \\ \hline & 1 & -8 & 32 & -112 & 432 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 0 & 16 & -16 \\ & & 2 & -4 & -8 & 16 \\ \hline & 1 & -2 & -4 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 16 & -16 \\ & & 4 & 0 & 0 & 64 \\ \hline & 1 & 0 & 0 & 16 & 48 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 16 & -16 \\ & & -1 & 5 & -5 & -11 \\ \hline & 1 & -5 & 5 & 11 & -27 \end{array}$$

Directions: Determine all properties of each polynomial function and sketch a graph WITHOUT a graphing calculator.

24.) $f(x) = x^3 + 3x^2 - 4x - 12$

P: 1, 2, 3, 4, 6, 12

Q: 1

a.) Determine the possible number of rational roots.

Possible Roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.) Determine the possible number of positive and negative real zeros.

$f(x) \rightarrow + \ + \ - \ -$ 1 CHANGE

$f(-x) \rightarrow - \ + \ + \ -$ 2 CHANGES

\# of possible positive zeros: 1

\# of possible negative zeros: 2 or 0

c.) Determine the linear factorization and zeros. Be sure to state if any zeros have multiplicity.

2	1	3	-4	-12
		2	10	12
	1	5	6	0

$x^2 + 5x + 6$

$(x+2)(x+3)$

OTHER POINTS ON GRAPH

1	1	3	-4	-12
		1	4	0
	1	4	0	-12

$\Rightarrow (1, -12)$

-4	1	3	-4	-12
		-4	4	0
	1	-1	0	-12

$\Rightarrow (-4, -12)$

-1	1	3	-4	-12
		-1	-2	6
	1	2	-6	-6

$\Rightarrow (-1, -6)$

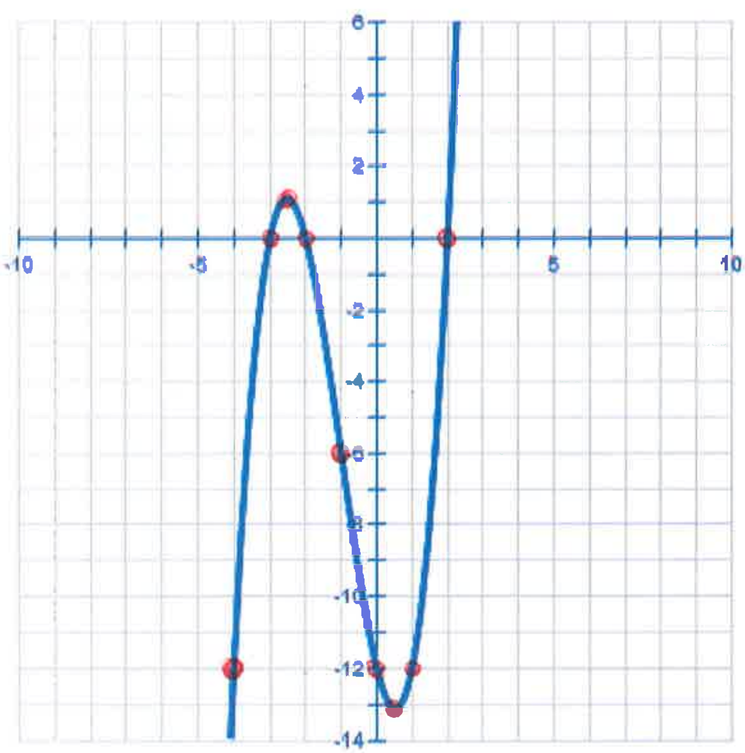
FACTORS: $(x-2)(x+2)(x+3)$

ZEROS: $x = \pm 2, \ x = -3$

d.) Determine the end behavior.

$x \rightarrow -\infty$ $f(x) \rightarrow$ $-\infty$

$x \rightarrow \infty$ $f(x) \rightarrow$ ∞



e.) Determine the possible number of turning points.

Max # of turning points: 2

f.) Determine the x-intercept(s).

$(-3, 0), (-2, 0), (2, 0)$

g.) Determine the y-intercept. $(0, -12)$

GIVEN: Maximum: $(-2.53, 1.13)$

GIVEN: Minimum: $(0.53, -13.13)$

25.) $f(x) = 2x^3 - 7x^2 - 5x + 4$

P: 1, 2, 4

Q: 1, 2

a.) Determine the possible number of rational roots.

Possible Roots: $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

b.) Determine the possible number of positive and negative real zeros.

$f(x) \rightarrow + \ - \ - \ +$ 2 CHANGES

$f(-x) \rightarrow - \ - \ + \ +$ 1 CHANGE

# of possible positive zeros: 2 OR 0

# of possible negative zeros: 1

c.) Determine the linear factorization and zeros. Be sure to state if any zeros have multiplicity.

$$\begin{array}{r|rrrr} -1 & 2 & -7 & -5 & 4 \\ & & -2 & 9 & -4 \\ \hline & 2 & -9 & 4 & 0 \end{array}$$

$2x^2 - 9x + 4$

$(2x - 1)(x - 4)$

OTHER POINTS ON GRAPH

$$\begin{array}{r|rrrr} 1 & 2 & -7 & -5 & 4 \\ & & 2 & -5 & -10 \\ \hline & 2 & -5 & -10 & -6 \end{array} \Rightarrow (1, -6)$$

$$\begin{array}{r|rrrr} 2 & 2 & -7 & -5 & 4 \\ & & 4 & -6 & -22 \\ \hline & 2 & -3 & -11 & -18 \end{array} \Rightarrow (2, -18)$$

$$\begin{array}{r|rrrr} 3 & 2 & -7 & -5 & 4 \\ & & 6 & -3 & -24 \\ \hline & 2 & -1 & -8 & -20 \end{array} \Rightarrow (3, -20)$$

FACTORS: $(x+1)(2x-1)(x-4)$

ZEROS: $x = -1, \frac{1}{2}, 4$

d.) Determine the end behavior.

$x \rightarrow -\infty$ $f(x) \rightarrow$ $-\infty$

$x \rightarrow \infty$ $f(x) \rightarrow$ ∞

e.) Determine the possible number of turning points.

Max # of turning points: 2

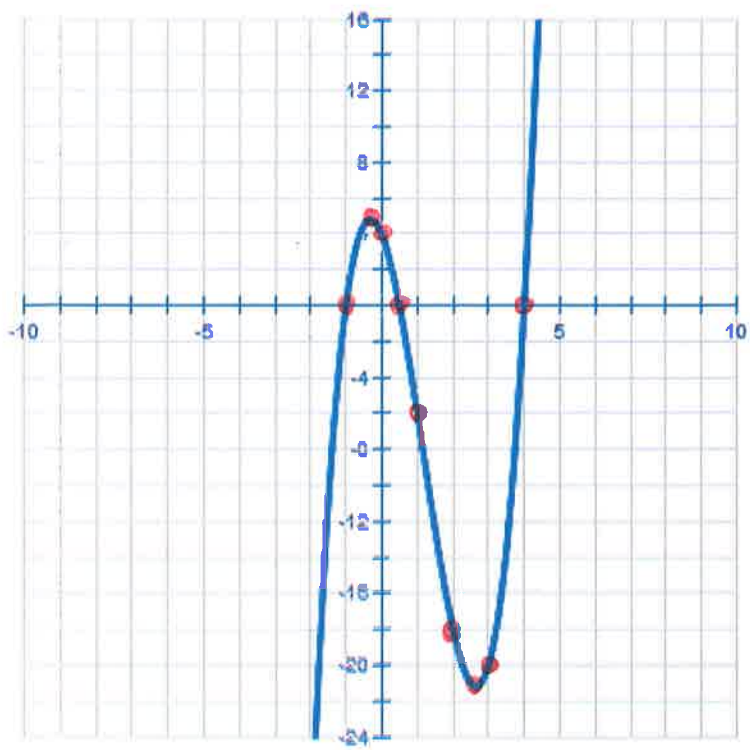
f.) Determine the x-intercept(s).

$(-1, 0), (\frac{1}{2}, 0), (4, 0)$

g.) Determine the y-intercept. $(0, 4)$

GIVEN: Maximum: $(-0.32, 4.82)$

GIVEN: Minimum: $(2.65, -21.19)$



26.) $f(x) = -x^4 + 4x^3 + x^2 - 16x + 12$

P: 1, 2, 3, 4, 6, 12

Q: 1

a.) Determine the possible number of rational roots.

Possible Roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.) Determine the possible number of positive and negative real zeros.

$f(x) \rightarrow - \quad + \quad + \quad - \quad +$ 3 CHANGES

$f(-x) \rightarrow - \quad - \quad + \quad + \quad +$ 1 CHANGE

# of possible positive zeros: 3 or 1

# of possible negative zeros: 1

c.) Determine the linear factorization and zeros. Be sure to state if any zeros have multiplicity.

1	-1	4	1	-16	12
		-1	3	4	-12
	-1	3	4	-12	0

2	-1	3	4	-12
		-2	2	12
	-1	1	6	0

$= -x^2 + x + 6$

$= -1(x^2 - x - 6)$

$= -(x-3)(x+2)$

OTHER POINTS ON GRAPH

-1	-1	4	1	-16	12
		1	-5	4	12
	-1	5	-4	-12	24

$\Rightarrow (-1, 24)$

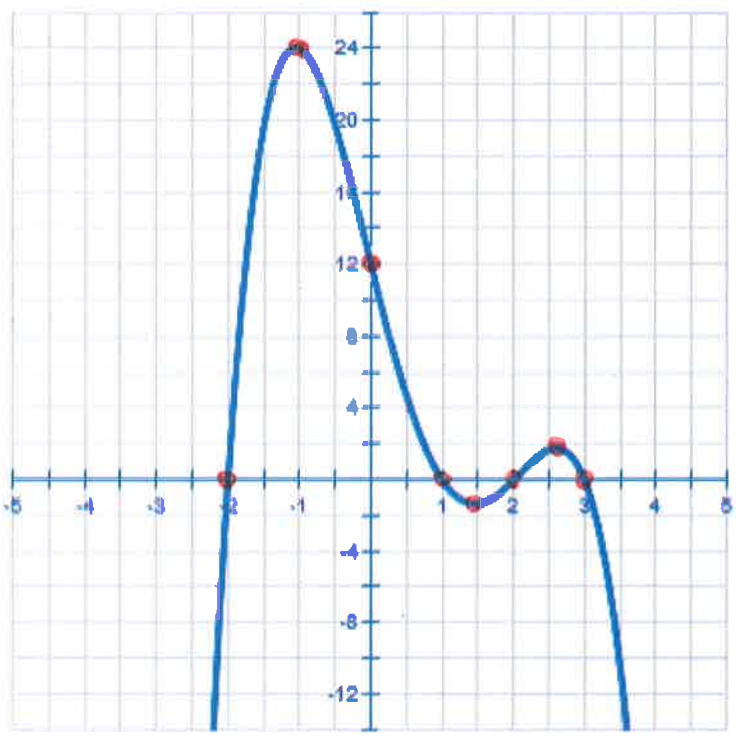
FACTORS: $-(x-1)(x-2)(x-3)(x+2)$

ZEROS: $x = -2, 1, 2, 3$

d.) Determine the end behavior.

$x \rightarrow -\infty$ $\quad f(x) \rightarrow$ $-\infty$

$x \rightarrow \infty$ $\quad f(x) \rightarrow$ $-\infty$



e.) Determine the possible number of turning points.

Max # of turning points: 3

f.) Determine the x-intercept(s).

$(-2, 0), (1, 0), (2, 0), (3, 0)$

g.) Determine the y-intercept. $(0, 12)$

GIVEN: Maximum: $(-1.06, 24.06)$ & $(2.60, 1.77)$

GIVEN: Minimum: $(1.46, -1.32)$

P: 1, 2, 4, 8, 16

Q: 1

27.) $f(x) = x^3 + 8x^2 + 20x + 16$

a.) Determine the possible number of rational roots.

Possible Roots: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

b.) Determine the possible number of positive and negative real zeros.

$f(x) \to + \quad + \quad + \quad +$ 0 CHANGES

$f(-x) \to - \quad + \quad - \quad +$ 3 CHANGES

\# of possible positive zeros: 0

\# of possible negative zeros: 3 OR 1

c.) Determine the linear factorization and zeros. Be sure to state if any zeros have multiplicity.

-2	1	8	20	16
		-2	-12	-16
	1	6	8	0

$x^2 + 6x + 8$

$(x+4)(x+2)$

OTHER POINTS ON GRAPH

-1	1	8	20	16
		-1	-7	
	1	7	13	

$\Rightarrow (-1, 3)$

-3	1	8	20	16
		-3	-15	-15
	1	5	5	1

$\Rightarrow (-3, 1)$

-5	1	8	20	16
		-5	-15	-25
	1	3	5	-9

$\Rightarrow (-5, -9)$

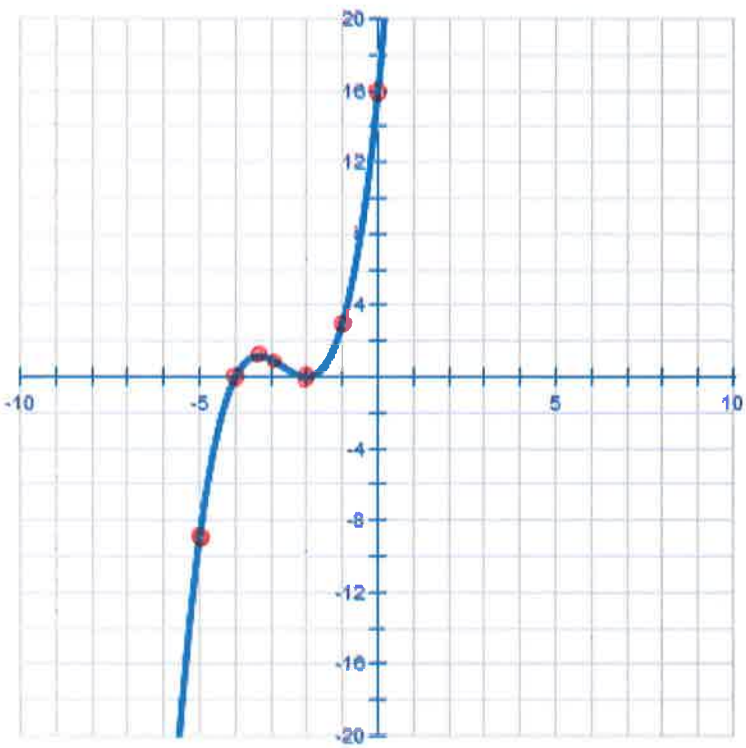
FACTORS: $(x+4)(x+2)^2$

ZEROS: $x = -4, -2$ (MULT OF 2)

d.) Determine the end behavior.

$x \to -\infty \qquad f(x) \to$ $-\infty$

$x \to \infty \qquad f(x) \to$ ∞



e.) Determine the possible number of turning points.

Max # of turning points: 2

f.) Determine the x-intercept(s).

$(-4, 0)\ (-2, 0)$

g.) Determine the y-intercept. $(0, 16)$

GIVEN: Maximum: $(-3.33, 1.19)$

P: 1, 2, 4, 8, 16

Q: 1, 3

28.) $f(x) = -3x^3 + 20x^2 - 36x + 16$

a.) Determine the possible number of rational roots.

Possible Roots: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{16}{3}$

b.) Determine the possible number of positive and negative real zeros.

$f(x) \rightarrow - \quad + \quad - \quad +$ 3 CHANGES

$f(-x) \rightarrow + \quad + \quad + \quad +$ 0 CHANGES

# of possible positive zeros: 3 OR 1

# of possible negative zeros: 0

c.) Determine the linear factorization and zeros. Be sure to state if any zeros have multiplicity.

OTHER POINTS ON GRAPH

$$\begin{array}{r|rrrr} 2 & -3 & 20 & -36 & 16 \\ & & -6 & 28 & -16 \\ \hline & -3 & 14 & -8 & 0 \end{array}$$

$= -3x^2 + 14x - 8$

$= -(3x^2 - 14x + 8)$

$= -[(3x^2 - 12x)(-2x + 8)]$

$= -[3x(x-4) - 2(x-4)]$

$= -(x-4)(3x-2)$

FACTORS: $-(x-2)(x-4)(3x-2)$

ZEROS: $x = 2, 4, \frac{2}{3}$

d.) Determine the end behavior.

$x \rightarrow -\infty \qquad f(x) \rightarrow$ ∞

$x \rightarrow \infty \qquad f(x) \rightarrow$ $-\infty$

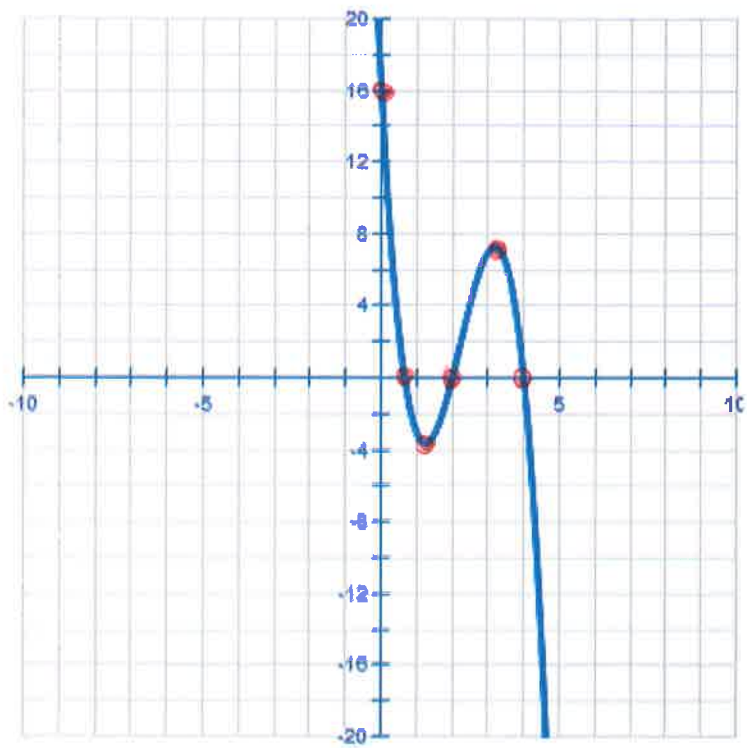
e.) Determine the possible number of turning points.

Max # of turning points: 2

f.) Determine the x-intercept(s).

$(\frac{2}{3}, 0), (2, 0), (4, 0)$

g.) Determine the y-intercept. $(0, 16)$



GIVEN: Maximum: $(3.19, 7.30)$

GIVEN: Minimum: $(1.25, -3.61)$

P: 1, 2, 3, 4, 6, 9, 12, 18, 36

Q: 1

29.) $f(x) = x^4 - 5x^2 - 36$

a.) Determine the possible number of rational roots.

Possible Roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

b.) Determine the possible number of positive and negative real zeros.

$f(x) \rightarrow + \; - \; -$ 1 CHANGE

$f(-x) \rightarrow + \; - \; -$ 1 CHANGE

\# of possible positive zeros: 1

\# of possible negative zeros: 1

c.) Determine the linear factorization and zeros. Be sure to state if any zeros have multiplicity.

3 | 1 0 -5 0 -36
3 9 12 36
1 3 4 12 | 0

-3 | 1 3 4 12
-3 0 -12
1 0 4 | 0

$x^2 + 4$

$(x+2i)(x-2i)$

OTHER POINTS ON GRAPH

1 | 1 0 -5 0 -36
1 1 -4 -4
1 1 -4 -4 | -40
$\Rightarrow (1, -40)$

-1 | 1 0 -5 0 -36
-1 1 4 -4
1 -1 -4 4 | -40
$\Rightarrow (-1, -40)$

2 | 1 0 -5 0 -36
2 4 -2 -4
1 2 -1 -2 | -40
$\Rightarrow (2, -40)$

-2 | 1 0 -5 0 -36
-2 4 2 -4
1 -2 -1 2 | -40
$\Rightarrow (-2, -40)$

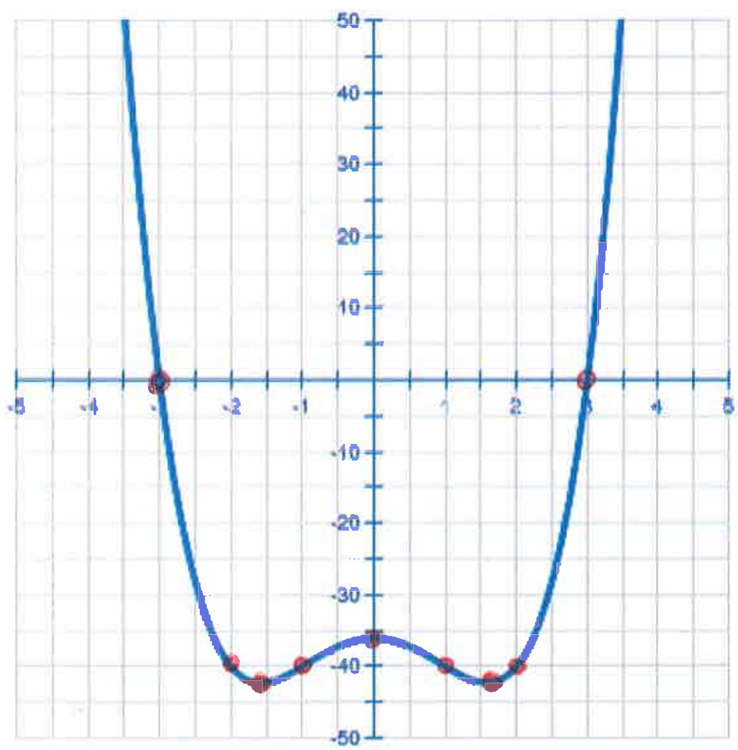
FACTORS: $(x-3)(x+3)(x-2i)(x+2i)$

ZEROS: $x = \pm 3$, $x = \pm 2i$

d.) Determine the end behavior.

$x \to -\infty$ $f(x) \to$ ∞

$x \to \infty$ $f(x) \to$ ∞



e.) Determine the possible number of turning points.

Max # of turning points: 3

f.) Determine the x-intercept(s).

$(3, 0)$ $(-3, 0)$

g.) Determine the y-intercept. $(0, -36)$

GIVEN: Maximum: $(0, -36)$

GIVEN: Minimum: $(-1.58, -42.25)$ & $(1.58, -42.25)$